Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2015

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 13 pages including 1 information sheet, and a SPECIAL ANSWERBOOK.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.

QUESTION 1

The data in the table below represents the marks obtained by 10 Grade 12 learners for English Home Language (HL) and Afrikaans First Additional Language (FAL).

English HL	42	54	85	32	63	71	92	62	58	66
Afrikaans FAL	50	58	80	45	60	65	98	75	71	58

1.1 Draw a scatter plot of the data above by making use of the grid provided in the SPECIAL ANSWER BOOK. (4)

1.2 Calculate the equation of the least squares regression line for this data. (3)

1.3 Calculate the correlation coefficient. (2)

1.4 Describe the correlation between English Home Language and Afrikaans First Additional Language. (1)

1.5 Predict the final English Home Language mark for the learner who obtained 74 marks in Afrikaans First Additional Language. (2)

[12]

QUESTION 2

The weights (in kilogram) of the 20 boys in the hockey squad of School A are given below:

69	59	59	66	64	58	63	58	62	61
57	53	60	51	60	48	47	60	40	60

2.1 Determine the mean and variance for the weights of the School A squad. (3)

2.2 The following information was obtained from the School B boys' hockey coach, regarding the weights of the boys in his squad.

$$\sum_{n=1}^{22} x_n = 1320 \text{ and } \sum_{n=1}^{22} (x_n - 60)^2 = 1012$$

2.2.1 How many boys are in the School B squad? (1)

2.2.2 Determine the mean weight for the School B squad. (2)

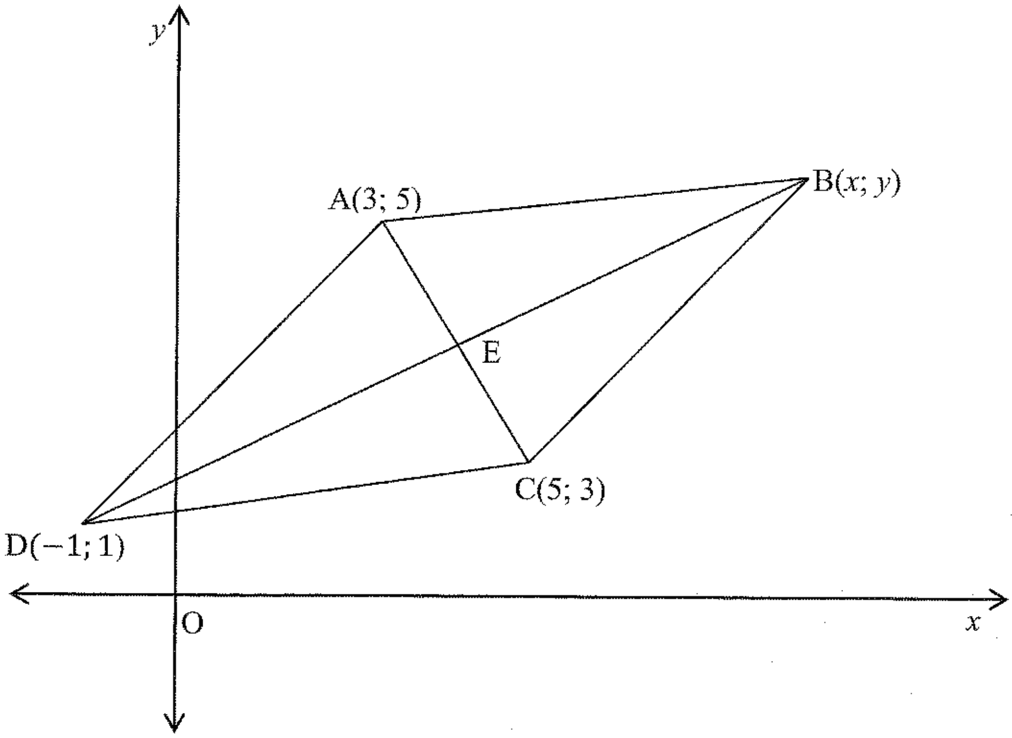
2.2.3 Determine the standard deviation for the School B squad. (2)

2.3 If five boys of equal weight are added to the squad of School A so that the means of both schools are the same, what must be the weight of each boy? (2)

[10]

QUESTION 3

In the figure A(3 ; 5), B(x ; y), C(5 ; 3) and D(−1; 1) are the vertices of parallelogram ABCD. AC and BD, the diagonals of the parallelogram, intersect at E.



3.1 Determine:

3.1.1 The co-ordinates of E (2)

3.1.2 The co-ordinates of B (3)

3.1.3 The co-ordinates of the midpoint F, of CD and hence the equation of the line passing through F, parallel to AD. (5)

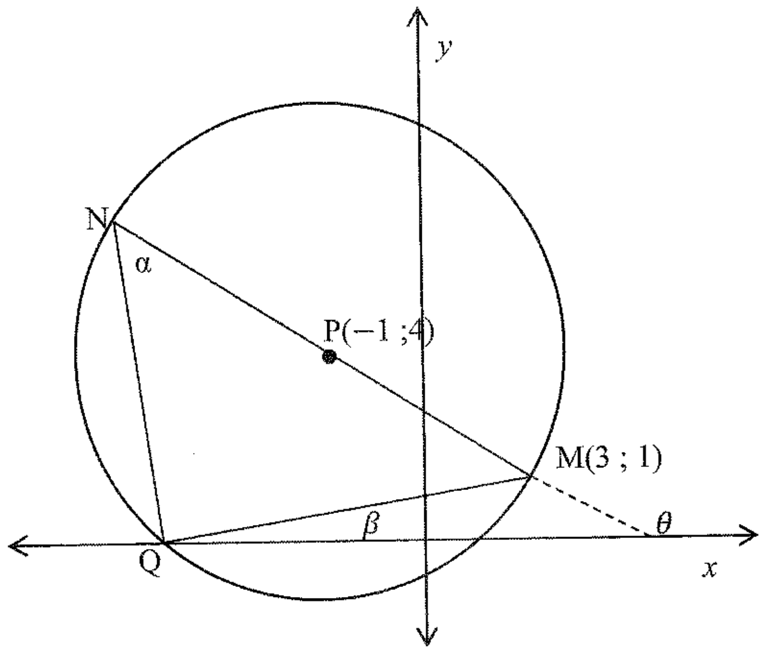
3.2 The points G(t +1 ; 2,5) ,D(−1; 1) and E(4 ; 4) and are collinear. Calculate the value of t. (4)

3.3 Determine, by calculations, whether ABCD is a rhombus or not. Give a reason for your answer. (5)

[19]

QUESTION 4

In the diagram below, M(3 ; 1), Q and N lie on the circumference of circle with centre P(−1 ; 4) and form ΔMQN. NPM is a straight line.



4.1 Determine the equation of the circle. (4)

4.2 Why is $N\hat{Q}M = 90°$? (1)

4.3 Show that the co-ordinates of Q are (−4; 0). (3)

4.4 Calculate the gradient of MN. (2)

4.5 Hence, calculate the size of α. (5)

4.6 Determine the equation of a tangent to the circle at M. (5)
[20]

QUESTION 5

5.1 Prove, **without the use of a calculator**, that,

$$\cos 75^\circ + \cos 15^\circ = \frac{\sqrt{6}}{2}$$ (4)

5.2 Determine the general solution of:

$$1 + 4\sin^2 x - 5\sin x + \cos 2x = 0$$ (7)

5.3 Prove the identity

$$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$ (3)

5.4 Simplify WDC :

$$\frac{\sin(450^\circ - x)\tan(x - 180^\circ)\sin 23^\circ \cos 23^\circ}{\cos 44^\circ \sin(-x)}$$

(6)

[20]

QUESTION 6

Given $f(x) = \sin(x - 30^\circ)$ and $g(x) = \cos 3x$ for $x \in [-90^\circ; 90^\circ]$

6.1 Write down the period of g. (1)

6.2 Use the set of axes provided in the SPECIAL ANSWER BOOK, to draw sketch graphs of f and g for $x \in [-90^\circ; 90^\circ]$. Clearly show all intercepts with the axes and the co-ordinates of all the turning points and end points of both curves. (6)

6.3 Use the graphs to determine the value(s) of x for $x \in [-90^\circ; 90^\circ]$, where:

6.3.1 $f(x) > g(x)$ (2)

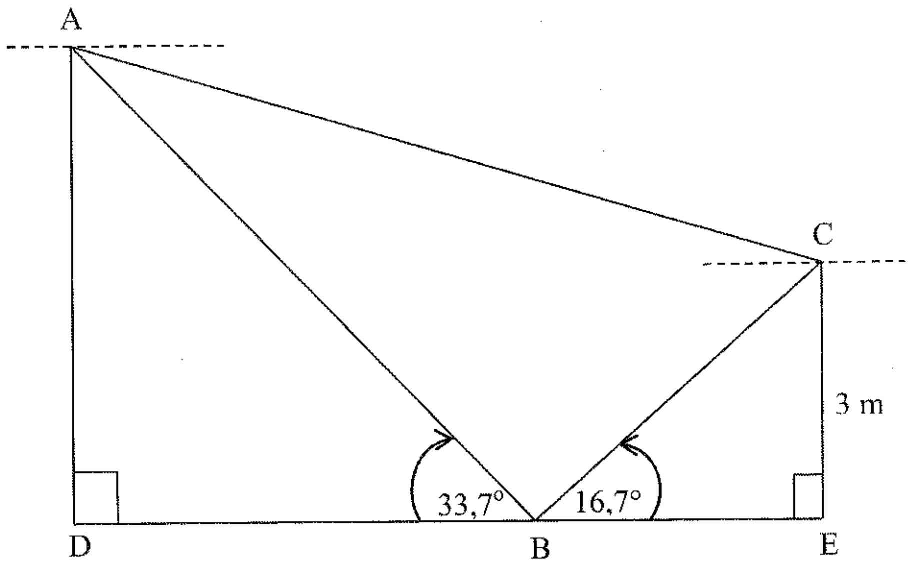
6.3.2 $f(x).g(x) > 0$ (2)

6.4 Determine the range of $h(x) = 3f(x) - 1$. (2)

[13]

QUESTION 7

In the diagram below, C is a point on one side of the Buffalo River and is 3 m above the water. A is a point on the other side of the river directly opposite C on the higher bank. B is a boat on the river. A, B and C are in the same vertical plane. The angle of depression of B from A is 33,7°. The angle of depression C from A is 15, 60° and B from C is 16,7°.



7.1 Calculate the length of BC. (3)

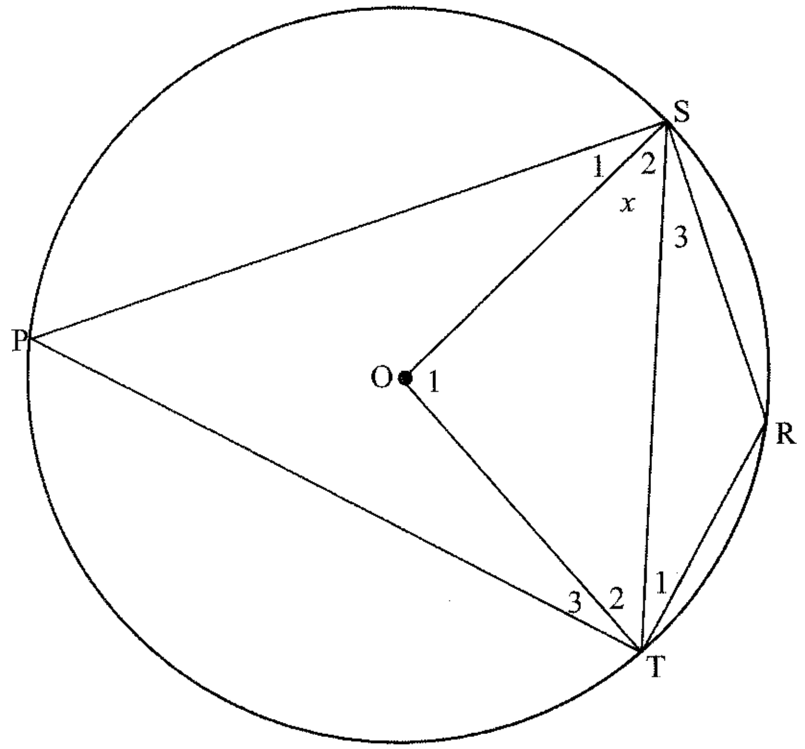
7.2 Calculate the length of AB. (3)

7.3 Calculate the length of AD. (3)
[9]

Give reasons for ALL statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram below, O is the centre of the circle which passes through P, T, R and S. PTRS is a cyclic quadrilateral and ST is drawn. $\hat{S}_2 = x$.



8.1 Express, giving reasons, each of the following angles in terms of x.

8.1.1 $\hat{O}_1$ (2)

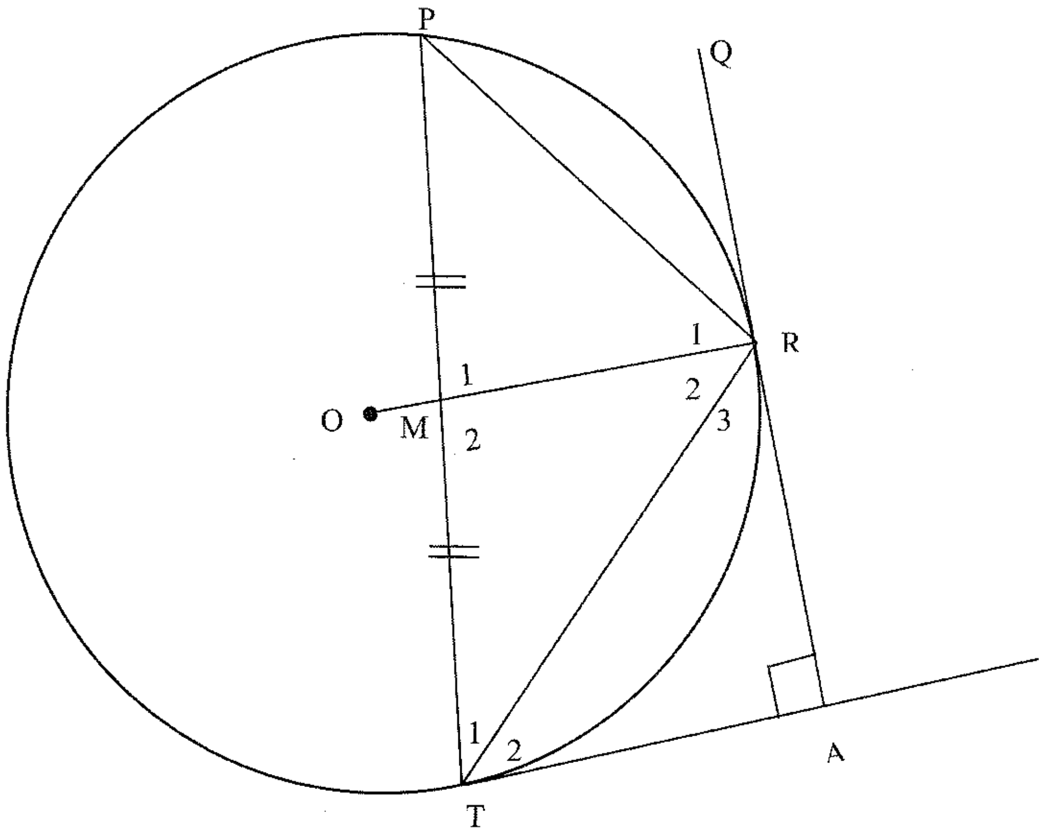
8.1.2 $\hat{P}$ (2)

8.1.3 $\hat{R}$ (2)

8.2 Hence, calculate the value of x if SOTR is a parallelogram. (3)
[9]

QUESTION 9

In the diagram below, M is the midpoint of chord PT of circle with centre O. OR is a radius passing through M. QR is produced to intersect tangent TA at A, such that TA ⊥ RA. T and R are joined.



Prove, stating reasons, that:

9.1 MTAR is a cyclic quadrilateral (4)

9.2 PR = TR (5)

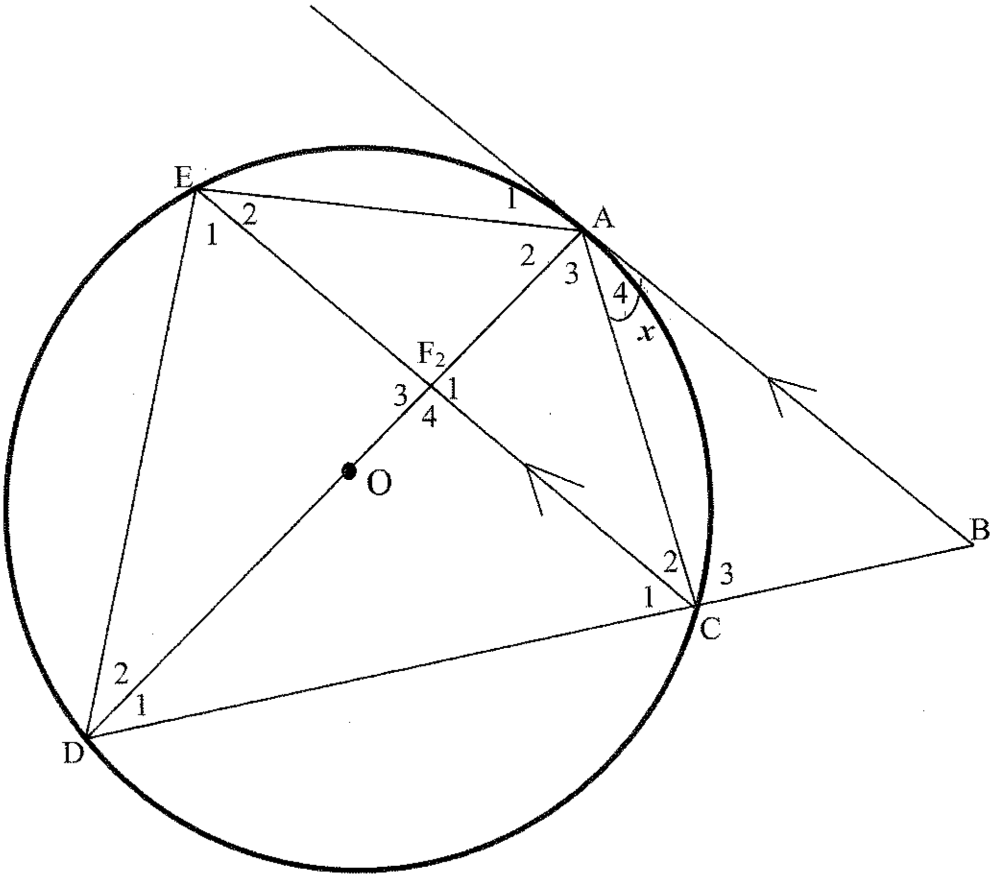
9.3 $\hat{T}_1 = \hat{T}_2$ (3)

[12]

QUESTION 10

10.1 Complete the statement of the following theorem:
If two triangles are equiangular then their corresponding sides are ... and the two triangles are similar. (1)

10.2 In the figure below, AB is a tangent to the circle with the centre O. AC = AO and BA ∥ CE. DC produced cuts tangent BA at B.



10.2.1 If $A_4 = x$, determine with reasons three other angles equal to x. (3)

10.2.2 Prove that ΔACF ||| ΔADC. (3)

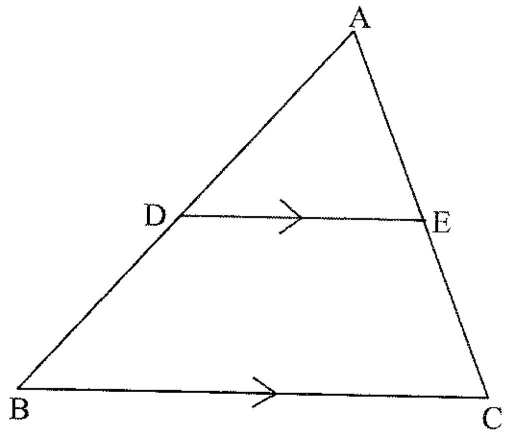
10.2.3 Prove that $AF^2 = \frac{AO^2}{AD}$ (4)

[11]

ie $AF = \frac{AO^2}{AD}$

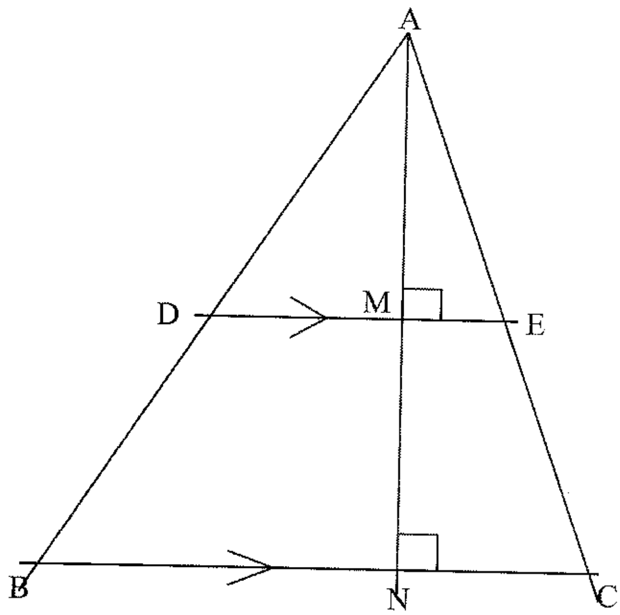
QUESTION 11

11.1 Make use of the diagram in the SPECIAL ANSWER BOOK, to prove the theorem which states that if DE ∥ BC then, $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

11.2 In the diagram below, DE ∥ BC, AN ⊥ DE and BC. $\frac{AD}{DB} = \frac{3}{2}$.



Write down the values of:

11.2.1 $\frac{AM}{MN}$ (2)

11.2.2 $\frac{DE}{BC}$ (4)

11.2.3 $\frac{\text{area } \Delta ADE}{\text{area } \Delta ABC}$ (3)

[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$